THE ENERGY LOSSES OF RELATIVISTIC HIGHCHARGED IONS

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Abstract

The energy losses of heavy multiplycharged ions at collisions with light atoms and polarization losses at moving through matter have been considered under circumstances the ion charge $Z \gg 1$ and the relative colliding velocity $v \gg 1$, so that $Z \sim v \leq c$, where c - the light velocity (atomic units are used). In this region of parameters the Born approximation is not applicable. The simple formulas for effective stopping are obtained. The comparison with other theoretical results and experiments are given.

1 Introduction

Usually an energy loss of a relativistic charged particle in collisions with an atom is calculated in the Born approximation which is applicable if $Z/v \ll 1$ (here Z - the ion charge, v - relative velocity of collision, atomic units are used) (Berestetskii et al 1989). However, there were some resent experiments which use ions of such a large charge so the above mentioned condition was violated even at $v \leq c \approx 137$ (see for instance Kelbch et al 1986, Berg et al 1988, Berg et al 1992, Scheidenberger et al 1994). The approximations applicable at $Z/v \sim 1$ - Eikonal approximation and its modifications (McGuire 1982, Crothers and McCann 1983), the Method of Sudden Perturbations (Eichler 1977, Salop and Eichler 1979, Yudin 1981, Yudin 1991), as well as the Method of the Classical Trajectories (Olson 1988) demand considerable numerical calculations even in nonrelativistic range of collision velocities. Relativistic collision velocities make the the calculations even more complicated (Yudin 1991).

In the present paper we consider the energy loss of heavy relativistic highcharged ions in collisions with light (nonrelativistic) atoms and polarizations losses at moving through the matter under conditions $Z \sim v \leq c$, $Z \gg 1$, $v \gg 1$ by following a simple approach (Matveev 1991) and its relativistic modification (Matveev and Musakhanov 1994). We are going to obtain simple formulas describing an effective stopping and compare it's results with the results of other theoretical methods and experiments.

2 Collisions with separated atom

Accordingly to (Landau and Lifshitz 1974) average losses of energy in collisions are characterized by the effective stopping

$$\kappa = \sum_n (\epsilon_n - \epsilon_0)\sigma_n \ . \tag{1}$$

Here ϵ_n and ϵ_0 are the energies of excited $|n>$ and bound $|0>$ atom's states, σ_n is the cross section for excitation of state $|n>$. First, we consider, for simplicity, the collision of a relativistic highcharged ion with a hydrogen atom. Accordingly to (Matveev 1991, Matveev and Musakhanov 1994) the whole interval $0 < b < \infty$ of all possible values of impact parameter b may be divided into three ranges:

$$A)\ 0 < b < b_1\,;\quad B)\ b_1 < b < b_0\,;\quad C)\ b_0 < b < \infty\,, \tag{2}$$

which correspondent to small, middle and large values of an impact parameter. The border amounts $b_1,\ b_0$ are (Matveev and Musakhanov 1994)

$$b_1 \sim 1\,;\quad b_0 \sim\ v/\sqrt{1-\beta^2}\,;\quad \beta = v/c\,.$$

We are going to calculate κ in each range from (2) and obtain total effective stopping by summing those values . The exact meaning of b_0 and b_1 are unimportant for us, since dependence κ upon sewing parameters $b_1,\ b_0$ appears to be logarithmic, which lead to the correct sewing of contributions from each region (2) and vanishing any dependence κ on b_1 and b_0 in the final result.

The range of small impact parameters is the range of large momentum transfers, thus we needn't take into account the interaction of an electron with an atom nuclei, and therefore we can consider the scattering of an ion on an electron having rested before collision (Berestetskii et al 1989 §82). In this case an effective stopping may be expressed through using $\sigma(\epsilon)$ which is a cross section for collision with energy transfer ϵ:

$$\kappa = \int_{\epsilon_{min}}^{\epsilon_{max}} \epsilon\sigma(\epsilon)d\epsilon \ . \tag{3}$$

Here we can't, unlike (Berestetskii et al 1989), use $\sigma(\epsilon)$ in the Born approximation, because a condition $Z/v \ll 1$ is invalid. To go on, we assume colliding ion to be an infinitely heavy particle, and consequently it doesn't change its moving. This case the cross section for scattering of an ion moving at fixed velocity on the free and motionless electron may be obtained by transformation to the system where ion is motionless. We mark θ a respective angle of scattering. Following to (Berestetskii et al 1989) we can maintain an energy transfer to be a function $\epsilon = \epsilon(\theta)$, except the case of a superhigh energy. That function has a form

$$\epsilon(\theta) = \frac{2v^2}{(1-\beta^2)} Sin^2\frac{\theta}{2} \ . \tag{4}$$

The values ϵ_{max} and ϵ_{min} from (3) are reached at $\theta = \pi$ and $\theta = \theta_{min}$ respectively. As a result we can rewrite (3) in a form

$$\kappa = 2\pi \frac{Z^2}{v^2} \int_{\theta_{min}}^{\pi} \frac{\sigma(\theta)}{\sigma_R(\theta)}\, ctg\frac{\theta}{2} d\theta \ , \tag{5}$$

Here $\sigma(\theta)$ is the cross section for the scattering of electron on motionless ion with a charge Z (at electron velocity v), which is obtained (Ahieser and Berestetskii 1969, Mott and Massey 1965) from exact solution of a scattering problem for the Dirac equation; σ_R is the Rutherford cross section:

$$\sigma_R = \frac{Z^2(1-\beta^2)}{c^4\beta^4(1-cos\theta)^2} \ . \tag{6}$$

The ratio $\sigma(\theta)/\sigma_R(\theta) \to 1$ at $\theta \to 0$ (Ahieser and Berestetskii 1969, Doggett and Spenser 1956), and hence in order to define θ_{min} we can use the quassiclassical connection (Landau and Lifshitz 1973) between a scattering angle and an impact parameter (see also the obvious qualitative picture of collision suggested in (Matveev 1991))

$$\theta_{min} = \frac{2Z}{v^2 b_1}\sqrt{1-\beta^2} \ . \tag{7}$$

One can easy see that the integral (5) depends on angle θ_{min} in logarithmic way (at small θ_{min}), thus the formula (5) may be written in a form:

$$\kappa = 4\pi \frac{Z^2}{v^2} \ln \frac{v^2 b_1}{Z\sqrt{1-\beta^2}\, a(Z,v)} \ . \tag{8}$$

The function a(Z,v) are defined by us from comparing (8) with the numerical calculation results by formula (5) with the ratio σ/σ_R from (Doggett and Spenser 1956). As a result we can approximate function a(Z,v) with a following formula:

$$a(Z,v) = (-0.23016 \cdot \alpha - (1.00832 \cdot \alpha - 0.32388)\beta^2 + 1)^2 \ , \tag{9}$$

here $\alpha = Z/c$. In the tables number 1 and number 2 the effective stopping calculated (at $b_1 = 1$) by formula (5) (table 1) and by formula (8) with substituting the function a(Z,v) from (9) (table 2) are brought. In these tables the data of 1st column are ion's energies (in Mev/nucleon) correspondent to the relative velocities from (Doggett and Spenser 1956); the data of next columns are κ values (in atomic units) for the ions with a charge: 6, 13, 29, 50, 82, 92 respectively. One can see from the tables that the supposed approximation is enough good, at any rate in the limits of v and Z which the data of (Doggett and Spenser 1956) are brought for. Besides we have to emphasize that function $a(Z,v) \longrightarrow 1$ at $\beta \longrightarrow 0$ (nonrelativistic limit) and $\alpha \longrightarrow 0$, which conforms to the fact that $\sigma/\sigma_R \longrightarrow 1$ at $\beta \longrightarrow 0$, $\alpha \longrightarrow 0$.

The collisions with the middle impact parameter $b_1 < b < b_0$ make the main contribution to the cross sections for the inelastic process (Matveev 1991, Matveev and Musakhanov 1994). The energy transfer in that collision is $\epsilon \leq 1 \sim$ ionization potential of an atom, and therefore the contribution to the effective stopping from that collision

Ion's energy	Ion's charges					
Mev/nucleon	6	13	29	50	82	92
91.8	0.894	3.714	16.247	44.368	110.067	135.497
183.6	0.554	2.333	10.411	29.185	75.125	93.469
367.2	0.372	1.580	7.160	20.452	53.402	68.540
734.4	0.280	1.200	5.495	15.893	43.418	55.109
1285.2	0.245	1.056	4.872	14.181	39.220	49.993
1836	0.235	1.015	4.697	13.705	38.063	48.618
3672	0.231	1.003	4.670	13.671	38.079	48.701
7344	0.239	1.043	4.879	14.310	40.246	50.941
18360	0.257	1.129	5.311	15.595	43.308	55.302

Table 1: Effective stoping (at.units) - result of numerical integration by formula (5) with using σ/σ_R from (Doggett and Spenser 1956)

Ion's energy	Ion's charges					
Mev/nucleon	6	13	29	50	82	92
91.8	0.891	3.697	16.056	43.666	110.134	137.246
183.6	0.552	2.319	10.268	28.454	73.567	92.369
367.2	0.370	1.570	7.066	19.922	52.838	66.916
734.4	0.278	1.192	5.439	15.559	42.255	53.985
1285.2	0.244	1.051	4.834	13.951	38.430	49.379
1836	0.234	1.010	4.666	13.520	37.462	48.252
3672	0.230	0.999	4.647	13.529	37.685	48.636
7344	0.238	1.040	4.860	14.185	39.523	50.991
18360	0.256	1.125	5.291	15.475	43.018	55.405

Table 2: Effective stoping (at.units), by formulas (8) with using a funstion a(Z,v) defined from (9)

can't be accounted by using a perturbation theory (Eichler 1977, Yudin 1981, Matveev 1991, Matveev and Musakhanov 1994). We must note else, that an atom electron is non-relativistic as before so after collision with an impact parameter from range $b_1 < b < b_0$ (Matveev and Musakhanov 1994). The contribution of a collision with such impact parameter to an effective stopping may be easy obtained from formula (1) by substituting to it the cross section for the inelastic process (Matveev and Musakhanov 1994)

$$\sigma_n = \int_{b_1}^{b_0} 2\pi b \,|< n \mid exp(-i\vec{q}\vec{r}) \mid 0 >|^2 \, db \,, \tag{10}$$

Here $\vec{q} = 2Z\vec{b}/(vb^2)$ and $\vec{b}$ is impact parameter vector, $\vec{r}$ is a coordinate of atomic electron. On repeating the calculations of (Landau and Lifshitz 1974) (but our case is simpler, because the upper limit in integration (10) doesn't depend on the final state of an atom) we have for an effective stopping:

$$\kappa = \sum_n (\epsilon_n - \epsilon_0)\sigma_n = 4\pi \frac{Z^2}{v^2} \ln \frac{q_1}{q_0} \,, \tag{11}$$

here $q_0 = 2Z/(vb_0\,); \quad q_1 = 2Z/(vb_1)$.

We need to account the contribution from collisions with an impact parameter belonging to the range $b_0 < b < \infty$. In this range the interaction between an ion and an atom may be accounted by using a perturbation theory (Eichler 1977, Matveev 1991, Matveev and Musakhanov 1994). Amplitude for the transition of an atom from a state $|0>$ to a state $|n>$ may be obtained following to (Moiseiwitsch 1985)

$$A_n = \frac{2iZ}{v^2} \Omega_n \vec{r}_{0n} \left[i\frac{\vec{v}}{v}(1-\beta^2)K_0(\xi) + \frac{\vec{b}}{b}\sqrt{1-\beta^2}\, K_1(\xi) \right] \,, \tag{12}$$

here $\Omega_n = \epsilon_n - \epsilon_0$; $\xi = \Omega_n b\sqrt{1-\beta^2}/v$; $K_0(\xi)$, $K_1(\xi)$ - the McDonald functions, $\vec{r}_{0n} =< 0|\vec{r}|n >$.

The cross section correspondent to (12) is

$$\sigma_n(b_0 < b < \infty) = \int d^2b \mid A_{0n} \mid^2 \ .$$

The integrating in this formula is made in limits: the angle of vector $\vec{b}$ changes from 0 to 2π and $b_0 < b < \infty$. In a result the cross section has a form [1]:

$$\sigma_n = 4\pi \frac{Z^2}{v^2} \mid x_{0n} \mid^2 \left(\ln \frac{4v^2}{\eta^2 b_0^2 \Omega_n^2 (1-\beta^2)} - \beta^2 \right) \,, \tag{13}$$

Here $\eta = e^B = 1.781$ (B=0.5772 - the Euler constant), $x_{0n} =< n|x|0 >$. The contribution to a effective stopping from collisions with large impact parameters may be calculated through substituting expression (13) to formula (1)

$$\kappa = 4\pi \frac{Z^2}{v^2} \left\{ \ln \frac{2v}{\eta I b_0 \sqrt{1-\beta^2}} - \beta^2/2 \right\} \,, \tag{14}$$

[1] To say rigorously, formula (13) was obtained from supposition that $\xi = \Omega_n b\sqrt{1-\beta^2}/v \ll 1$ ($\Omega_n \sim 1$), then $b \ll v/\sqrt{1-\beta^2} \sim b_0$, so following sewing may be performed exactly at such b.

Here we have introduced "average atom energy"- I (Berestetskii et al 1989 §82), which is

$$\ln I = \frac{\sum_n (\epsilon_n - \epsilon_0) \mid x_{0n} \mid^2 \ln(\epsilon_n - \epsilon_0)}{\sum_n (\epsilon_n - \epsilon_0) \mid x_{0n} \mid^2} \tag{15}$$

The total effective stopping of a relativistic highcharged ion on a hydrogen atom is obtained on summing the results of formulae (14),(11) and (8):

$$\kappa = 4\pi \frac{Z^2}{v^2} \left\{ \ln \frac{2v^3}{\eta I Z (1-\beta^2) a(Z,v)} - \beta^2/2 \right\} . \tag{16}$$

We would like to bring κ value calculated in the Born approximation from (Berestetskii et al 1989)

$$\kappa = 4\pi \frac{Z^2}{v^2} \left\{ \ln \frac{2v^2}{I(1-\beta^2)} - \beta^2 \right\} . \tag{17}$$

It should be noticed that nonrelativistic limit for ionization losses (16) (in accounting of $a(Z,v) \to 1$ at $\beta \to 0$ and $\alpha \to 0$)

$$\kappa = 4\pi \frac{Z^2}{v^2} \ln \frac{2v^3}{I\eta Z} \tag{18}$$

has a form which is similar to wellknown Bohr formula (Bohr 1913) obtained from classical representations. Besides we have to notice that our result (16) obtained on the basis of approach (Matveev 1991, Matveev and Musakhanov 1994), which is valid at $Z \sim v \gg 1$ only, doesn't have a way to pass (so does the Bohr formula) to the Born approximation which is valid at $v/Z \ll 1$.

As followed from our way of obtaining formula (16), in order to generalize it as well as the Born losses (17) to the case of collisions with multielectron atoms (whose electrons have velocities $v_a \ll v$, v is ion's velocity) we need to multiply the right part of formula (16) on Z_a (Z_a is the number of atom electrons) and change value I for average atom potential I_a defined, as before, from (15), but with values ϵ_n, ϵ_0 and $\mid x_{0n} \mid^2$ calculated for complex atom.

3 Energy losses in matter

Let's consider an energy loss of relativistic highcharged ion moving through matter. This losses are the sum of macroscopic (polarization) losses and losses at collision with separated atoms. The ion velocity hold to be much bigger than typical velocities of atomic electrons or at least the majority of them. Accordingly to (Fermi 1940) polarization losses of a charged particle moving in the matter are defined by the flow of the energy of electromagnetic field of a particle through the cylinder with radius b_0' built around a particle trajectory. The effective stopping is obtained by dividing of the flow on a particle velocity:

$$\kappa = \frac{Z^2 b_0'}{\pi v^2} \int_{-\infty}^{+\infty} d\omega K_0(b_0'\xi) K_1(b_0'\xi^*) \left(\frac{1}{\varepsilon(\omega)} - \beta^2 \right) i\omega\xi^* , \tag{19}$$

here $\xi^2 = \omega^2(v^{-2} - c^{-2}\varepsilon(\omega))$, $\varepsilon(\omega)$ - dielectrical penetration coefficient. At enough small cylinder radius b_0', i.e.

$$| b_0'\xi | \ll 1 \tag{20}$$

we have

$$\kappa = \frac{iZ^2}{\pi v^2}\int_{-\infty}^{+\infty}\omega d\omega \left\{\frac{1}{\varepsilon(\omega)} - \beta^2\right\}\ln\frac{2}{\eta b_0'\xi}\ , \tag{21}$$

where η=1.781 (like (13)). Other side, accordingly to (Landau and Lifshitz 1982) energy loss may be counted as a work of electromagnetic field per unit of way

$$\kappa = \frac{iZ^2}{\pi}\int_0^{q_0}\int_{-\infty}^{+\infty}\omega d\omega q dq \frac{(v^{-2} - c^{-2}\varepsilon(\omega))}{\varepsilon(\omega)(q^2 + \xi^2)}\ . \tag{22}$$

Formula (21) will coincide with (22) if condition (20) is valid and $q_0 = 2/(\eta b_0')$: surely,

$$\ln\frac{2}{\eta\xi b_0'} \approx \frac{1}{2}\ln\left(\frac{2^2}{\eta^2\xi^2 b_0'^2} + 1\right) = \int_0^{q_0}\frac{q dq}{q^2 + \xi^2}\ . \tag{23}$$

There are two distinguished cases to be considered separately: a) $v^2 < c^2/\varepsilon_0$ ($\varepsilon_0 = \varepsilon(0)$) - dielectrical penetration coefficient in static field) and b) $v^2 > c^2/\varepsilon_0$. In first case (Landau and Lifshitz 1982):

$$\kappa = 4\pi\frac{NZ^2}{v^2}\left[\ln\frac{q_0 v}{\bar{\omega}\sqrt{1-\beta^2}} - \frac{\beta^2}{2}\right]\ , q_0 = 2/(\eta b_0')\ . \tag{24}$$

Here $\bar{\omega}$ is a average amount of frequency of atom's electrons motion:

$$\ln\bar{\omega} = \frac{\int_0^\infty \omega\eta''(\omega)\ln\omega\, d\omega}{\int_0^\infty \omega\eta''(\omega) d\omega}\ ,$$

here $\eta'' = Im(\varepsilon^{-1}(\omega))$. In second case $((v^2 > c^2/\varepsilon_0))$: 1) if the particle energy is not rather large (motion energy is either less than or order of a rest energy of ion) we can use the formula (23); 2) In ultrarelativistic case (Landau and Lifshitz 1982)

$$\kappa = 2\pi\frac{NZ^2}{c^2}\ln\frac{q_0^2 c^2}{4\pi N}\ . \tag{25}$$

Now we must sum the macroscopic losses and the energy losses on separated atoms. In order to do it rewrite the condition (20) in a form:

$$b_0' \ll v/\ | \omega\sqrt{1-\beta^2\varepsilon}\ | < v/\sqrt{1-\beta^2} \sim b_0\ ,$$

where, for estimations, was assumed that $\omega \sim \omega_a \sim 1$ - typical atomic frequency. The condition of macroscopic method's being applicable is $b_0' \gg b_1 \sim 1$ - typical atomic size. Thus the lowest border b_0' is between b_1 and b_0:

$$b_1 \ll b_0' \ll b_0 \tag{26}$$

On comparing this condition with (2) we conclude that in order to obtain the total energy losses of relativistic highcharged ion moving through the matter we have to sum the contribution from ranges A and B from (2) (in range B the upper limit is equal to

projectile	target	calculation (*)		Our results	experiment (*)
$^{86}_{36}$Kr 900 MeV/u ($\beta = 0.861$)	Be	2.346	2.438	2.55200	2.432±0.037
$^{136}_{54}$Xe	Be	5.488	5.812	5.86187	5.861±0.076
780 MeV/u	C	6.014	6.378	6.43478	6.524±0.084
($\beta = 0.839$)	Al	5.404	5.755	5.80985	5.806±0.121
	Cu	4.703	5.036	5.08741	5.077±0.066
	Pb	3.654	3.942	3.98736	3.959±0.063

Table 3: Experimental and theoretical meanings of energy losses (in $Mev/(mg/cm^2)$) for various combinations target-ion.
(*)-Scheidenberger et al 1994

b_0') and polarization loss. The sum of contribution from ranges A and B according to (8) and (11) is equal

$$\kappa = 4\pi \frac{Z^2 N}{v^2} \ln \frac{v^2 b_0'}{Z\, a(Z,v)\sqrt{1-\beta^2}} \,. \tag{27}$$

We changed the number of atom electrons - Z_a for the number of electron in the unit of volume - N (as was hold in Landau and Lifsitz 1982). By summing (23) and (26) we obtain the total energy loss of relativistic highcharged ion moving through matter for the case $v^2 < c^2/\varepsilon_0$

$$\kappa = 4\pi \frac{Z^2 N}{v^2} \left[\ln \frac{2v^3}{Z\eta(1-\beta^2)a(Z,v)\bar{\omega}} - \beta^2/2 \right] . \tag{28}$$

As was said this formula is used often for the case $v^2 > c^2/\varepsilon_0$ and for the particle whose velocity is not too high. It should be noticed that (22) is different from (18) describing the energy losses on separated atoms with changing average potential I for $\bar{\omega}$ only (there is the same situation in Landau and Lifshitz 1982). In ultrarelativistic case on doing the same way as before we obtain total effective stopping in a form:

$$\kappa = 2\pi \frac{Z^2 N}{c^2} \left[\ln \frac{4c^6}{Z^2\eta^2(1-\beta^2)^2 a^2(Z,v) 4\pi N} - 1/2 \right] . \tag{29}$$

One can see that in this case accounting the polarization loss leads to slower growing an effective stopping power with increasing a collision velocity, in comparing with case of losses on separated atoms (18). One may rewrite the formula (29) introducing the Fermi corrections on density effect:

$$\kappa = 4\pi \frac{Z^2 N}{v^2} \left[\ln \frac{2v^3}{Z\eta(1-\beta^2)a(Z,v)I_a} - \beta^2/2 - \delta/2 \right] , \tag{30}$$

here the meaning of average ionization potential I_a and the Fermi corrections $\delta/2$ are brought from (Sternheimer et al 1984, Inokuti and Smith 1982).

In table 3 the theoretical and experimental meaning of effective stopping (in $Mev/(mg/cm^2)$) for various ions (we limited ourselves with Kr and Xe which have

enough large charge) and targets from (Scheidenberger et al 1994), as well as our results are brought: the column 1 - ion, it's energy (in Mev/nucleon) and amount $\beta = v/c$; the column 2 - the target's nature; the column 3 - the results of calculations (Scheidenberger et al 1994) by the Bete formula with the Fermi corrections; the column 4 - the results of calculating (Scheidenberger et al 1994) by the Bete formula with accounting Fermi corrections, Mott corrections (Mott 1929) and Bloch corrections (Bloch 1933); column 5 - are our results; column 6 - the experimental meaning for effective stopping (Scheidenberger et al 1994). One can see that our results are in rather good accordance with experiment.

4 Conclusion

The simple approach suggested in the papers (Matveev 1991, Matveev and Musakhanov 1994) enable us to estimate an effective stopping of relativistic highcharged ion in collisions with separated atoms and at motion in matter for many practically important cases, since the formulas obtained in this paper enable us to use the wellknown method of phenomenological corrections usually used in applied calculation. The region of applicability of our formulas $Z \sim v \leq c$ doesn't permit to make direct transition to the Born approximation ($Z/v \ll 1$). But that fact is not too significant limitation, because the region of applicability of the Born approximation for the ion with enough large charge (for example $Z = 92$) is unreachable even at $v \approx c$.

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